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COLLISIONAL RELAXATION OF THE NON-MAXWELLIAN PLASMA DISTRIBUTION IN A POLYWELL[tm]

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I. INTRODUCTION AND SUMMARY

The Polywell™ is a magnetic version of a Spherically Convergent Ion Focus (SCIF) device which was proposed by R. W. Bussard[1,2] as a significant variation of earlier studies on electrostatic confinement.[3,4] The idea of this device is to inject high energy electrons into a quasi-spherical magnetic field; the electrons create a potential well of sufficient depth to accelerate ions from low energy at the periphery to fusion energies within a focus at the center of the sphere. Injection of electrons keeps the system electrically <u>nonneutral</u>, so that the potential well which accelerates the ions is maintained at a constant value sufficient to confine the ions within the device, returning them again and again at high velocity to the central focus. Essential to the success of the scheme is that the ions maintain their nonthermal velocity distribution, with primarily radial flow, long enough to produce fusion in the dense focus at the center of the sphere.

If the ion source is at the periphery and ions return to their birth point (turning points), then an isotropic source at R will clearly produce an anisotropic (non-Maxwellian) distribution at $r < R$, since v_r increases as $\sqrt{2e\phi(r)/m_i}$, while from conservation of angular momentum $v_\perp$ increases as $v_\perp = v_{\perp R} R/r$, with $2e\phi_{max}/m_i = v_{\perp R}^2 R^2/r_c^2$ determining r_c, the closest approach of an ion to the center, where $v_{\perp R}$ is the velocity perpendicular to $\hat{i}_r$ of an individual ion at $r = R$, and where $\phi = 0$ at the location of the ion source.[1,2]

If the ions must converge to a high density, small radius focus, to form a reactor relevant configuration, ion-ion collisional effects can destroy the configuration, even if they do not cause ions to leave the device. There are two distinct effects that collisions have on ion convergence. First, as ions born at low energy pass through the dense core of the device, they will be nearly monoenergetic, at the energy of the well. Collisional processes will tend to thermalize the velocity distribution in the core. An ion which is upscattered in energy will have a turning point at a radius larger than the radius at which it was born and can eventually be lost to the dense core by escape from the device or by magnetic deflection, since B increases with radius in the multicusp field characteristic of SCIF devices, $B \sim r^m$, with m typically ≥ 3.

The second collisional effect involves isotropization of the ion velocity distribution in the bulk of lower density plasma outside the core. In this large scale region, ion-ion collisions can transfer momentum from the primarily radial flow, which produces good spherical convergence, to a more isotropic distribution of velocities. This deflection by collisions leads to an increase of local azimuthal velocity in this bulk region, which clearly decreases the convergence of the ions to the core.

In addition to the core and the bulk plasma which surrounds it, there is a third region with distinctive collisional properties, namely the edge plasma, where the bulk of the ions have their turning points. In this region ion velocities are low, and collisions much more frequent than in the interior. In this region the ion distribution due to previous collisions in the interior tends to become more anisotropic, while collisions in the edge itself make the edge distribution more isotropic. An isotropic distribution at the edge will produce exactly the strong radial convergence required for the SCIF. Thus due to the large ion orbits, collisional processes in the edge will tend to drive the distribution in the interior to stay anisotropic and convergent. In this, edge collisions compete with local collisions in the interior which attempt to produce a local Maxwellian. If edge collisions dominate, even though large angle scattering will eventually thermalize the entire system, the more rapid small angle scattering (which competes with the fusion reacting time scale) will force the distribution to stay non-Maxwellian in the interior. This effect is simply a matter of competitive rates of thermalization in the various regions.

In Section II, we calculate the ion collision frequencies in the various spatial regions, for both experimental and reactor grade plasmas. In Section III, we use a diffusion approximation to give heuristic estimates of the time scales for "loss" of ion convergence due to ion-ion collisional effects in the core and bulk of the device, as discussed above. In Section IV, we give a Fokker-Planck-like calculation of the ion distribution which results from the combined effect of collisions in all three regions, and show that the edge effects dominate, producing a Maxwellian edge and a non-Maxwellian convergent interior. In Section V we apply the results to estimating loss rates in a SCIF reactor.

II. ESTIMATES OF ION COLLISION RATES

In this section we estimate the various ion collision rates, as discussed in Section I. Because collisions depend on energy and density, we first present the radial density and energy profiles expected in an idealized version of the SCIF, and discuss the collisional rates in the different regions, for both experimental and reactor-grade parameters, using the parameter set in Table I.

A. ION ENERGY AND DENSITY PROFILES

As a first approximation for the motion of a test ion in a SCIF device, we assume the ion undergoes conservative motion in a spherically symmetric electrostatic potential well. The ion is born with an energy E_0 at a location R. The total energy of the particle is conserved:

$$E_i(r) = \frac{1}{2} m_i \, (v_r^2 + v_\perp^2) - e\phi(r) \quad . \tag{1}$$

Here E_i is the total energy, m_i is the ion mass, v_r and $v_\perp$ are the radial and azimuthal ion velocities, $v_\perp^2 = v_\phi^2 + v_\psi^2$ in a spherical coordinate system, and $\phi(r)$ is the electrostatic potential energy.

We assume the electrostatic potential has the following spatial form:

$$\phi(r) = \phi_{max} \left[1 - (\tfrac{r}{R})^p\right] \quad , \tag{2}$$

where ϕ_{max} is the value of the potential at $r = 0$, the center of the device, and $\phi = 0$ at $r = R$, where R is the location of the ion source. The actual form of $\phi(r)$ is not relevant to the qualitative results we obtain.

We assume that the ion is born at a location where magnetic effects are weak.[1,2] If this is not the case, the effect of collisions will be irrelevant, since we will show that collisional time scales are larger than the time it takes for an ion to transit the device, while magnetic effects would destroy convergence before a single pass. The magnetic deflection can be estimated by calculating the orbits of an ion in a magnetic field $B = B_0(r/R_B)^m$, where R_B is radial location where $B = B_0$, and a radial potential $\phi(r)$, taking the direction of B to be perpendicular to $\nabla\phi$ in order to find the maximum deflection. This was done in Reference 2, leading to a spatial spread in the ion focus due to magnetic deflection from the fields at $r < r_0$ of

$$\frac{\delta r}{R} \simeq \frac{\omega_{ci}^2}{(m + 1)^2} \left(\frac{r_0}{R_B}\right)^{2m} \frac{r_0^2}{(2e\phi_0/M)} \quad ,$$

where r_0 is the ion birthpoint ($r_0 < R_B$) and ϕ_0 the potential drop from $r = 0$ to $r = r_0$. For the purpose of calculating collisional effects, we assume that the source is at an r_0 which provides satisfactory convergence.

The test ion radial energy profile is then approximately

$$E_r = e\phi_{max} \left[1 - (\tfrac{r}{R})^p\right] + E_0 \quad . \tag{3}$$

The ion azimuthal energy profile follows from conservation of angular momentum, $rv_\perp(r)$ = constant, leading to

$$E_\perp = \frac{1}{2} m_i v_{\perp R}^2 \left(\frac{R}{r}\right)^2 \quad ,$$

where $v_{\perp R}$ is the ion's azimuthal velocity at birth.

We assume that the ion source is active for a time long compared with the time it takes for an ion to flow from $r = R$ to $r \approx 0$ and back again, and that the source produces a low energy ion cloud at $r = R$. The ion distribution everywhere, and the radial ion density profile, are then determined by the potential $\phi(r)$ and the ion distribution function f_i at the birth point, which can be written in terms of the constants of the motion (E and $rv_\perp$), since the Vlasov equation for the steady state is f_i $(E, rv_\perp)$ = constant. There are three quite different regions of the device, characterized by the relative values of the ion's birth energy, and azimuthal and radial energies. Within a very small radial distance inward from the ion birth point, the potential change is negligible, the ion energy is of the order of it's birth energy, and the density is given by $n = n_{edge}$. The potential increases inward radially, and since the maximum well depth is in the keV range, compared with birth energies in the several eV range, it follows that $2e\phi >> E_0$ a short radial distance from the edge, while $m_i v_{\perp R}^2 (R/r)^2 < 2e\phi$. These conditions hold for a large radial region of the device which we refer to as the bulk. Eventually, however, at a smaller radius, $r << R$, $m_i v_{\perp R}^2 (R/r_c)^2 = 2e\phi_{max}$ and the ions can come no closer to $r = 0$. This defines the dense core region. This inward motion clearly evolves a Maxwellian source at $r = R$ into a radially counterstreaming bi-Maxwellian f_i at $r < R$ proportional to

$$e^{-\frac{m_i}{2T_r}(v_r \pm v_d)^2} \quad e^{-\frac{m_i v_\perp^2}{2T_\perp(r)}} \quad ,$$

where the radial drift responds to the radial electric field, $v_d = (2e\phi/m_i)^{1/2}$, and the transverse "temperature" $T_\perp = m_i v_{\perp o}^2 (R^2/r^2)$ increases inwardly to provide conservation of angular momentum, with $m_i v_{\perp o}^2$ an average perpendicular energy of ions born at $r = R$. If the ion source is distributed rather than sharply localized, the same arguments presented here can be used with reference to azimuthal "slices" of the ion source.

Integrating f_i over velocity yields the density in the various radial regions of the device. As a model for the ion distribution at the birth point, we assume that at birth the ion distribution function is uniform in energy up to some small energy E_o, and uniform in angular momentum up to some small azimuthal velocity $v_{\perp o}$, and take the potential at the birth point to be $\phi = 0$. This distribution is described in terms of the constants of the motion ($E = (1/2)m_i(v_r^2 + v_\perp^2) - e\phi$, $rv_\perp$) by the function

$$f_i = 3(m_i/E_o)^{3/2}\, n_{edge} \quad , \quad 0 < E < E_o \quad , \quad 0 < rv_\perp < R\, v_{\perp o}$$

$$= 0 \quad \text{else} \quad .$$

A Maxwellian gives similar results for $n(r)$, but the flat distribution is easier to work with analytically. The Vlasov equation for steady state is $f(E, rv_\perp) =$ constant, so the density and distribution everywhere is determined by the potential and the value of f at the ion birth point. There are three regions. For a <u>very</u> small distance from the birth point, the potential is negligible, and the density is given by $n = n_{edge}$. In the bulk of the plasma, interior from the edge, where $E_o \ll 2e\phi$, and $m_i v_{\perp o}^2 (R/r)^2 < 2e\phi$, the density is given by integrating f over $v_\perp dv_\perp dv_r$ with the result (r_c is defined in Eq. (5) below)

$$n_B = \frac{3}{4} \left(\frac{E_o}{e\phi}\right)^{1/2} \left(\frac{R^2}{r^2}\right) n_{edge} \quad , \qquad r_c < r < R \quad . \tag{4}$$

Assuming that the potential reaches its full value at a moderately large distance from $r = 0$, we see that a mean electron density throughout the bulk of the device will go as $1/r^2$. Moving inward, the $(R/r)^2$ factor eventually becomes substantial, so that the density becomes much larger than n_{edge}. Eventually, the radius is so small that $m_i v_{\perp o}^2 (R/r)^2 > 2e\phi_{max}$. Inside that radius the density is changing fairly slowly, on the scale of the changing potential. This defines the radius of the dense core r_c as the radius at which $2e\phi = m_i v_{\perp o}^2 (R/r)^2$, because outside that radius $n \sim 1/r^2$, while inside that radius n is nearly constant, found by integrating f_i over velocity,

$$n_c \sim \left(\frac{E_o}{e\phi_{max}}\right)^{1/2} \left(\frac{R}{r_c}\right)^2 n_{edge} \quad , \qquad r < r_c \quad ,$$

$$r_c \approx (m_i v_{\perp o}^2 / 2e\phi_{max})^{1/2} R \quad . \tag{5}$$

The size of the core and the central density are seen to depend on the angular momentum at the outer turning points of the ion motion. This leads to an overall view of the SCIF which determines the density and energy profiles needed to calculate the collisional relaxation of f_i toward Maxwellian.

B. COLLISION FREQUENCY PROFILE

Using the ion distribution function discussed in the last section, where most of the ions have turning points $v_r = 0$ at the same radius $r = R$ at the periphery of the central potential $\phi(r)$, and taking the radial and azimuthal energy moments of the distribution in the three regions of the device, we find that the core and the edge are isotropic, while the bulk of the plasma is highly anisotropic, with $E_r \gg E_\perp$.

In the core, defined by Eq. (5), the ion density is relatively flat; using the above estimates for the ion energy and density, we write the ion-ion (hydrogen) collision time in the core, τ_{iic}, as (energies are in eV, density in cm^{-3})

$$\tau_{iic} = \frac{2 \times 10^7 \, (e\phi_{max})^{3/2}}{n_c \, \ln\Lambda_c} \quad , \tag{6}$$

where the Coulomb logarithm in the core is

$$\ln\Lambda_c \approx 7 + 2.3 \log_{10} \left[\frac{(e\phi_{max})^{3/2}}{(n_c/10^{14}\ cm^{-3})^{1/2}} \right] \quad .$$

Equation (6) gives $\tau_{iic} \cong 1$ s and 30×10^{-6} s, respectively, for the EXP and REACTOR parameters in Table I. In addition, $\ln\Lambda_c = 23$ and 20 for the EXP and REACTOR parameters.

In the bulk, the ion density decreases radially outward basically as $1/r^2$, with an additional weak radial dependence from ϕ, which tends to make the density distribution flatter than $1/r^2$ beyond about the R/2 point of the device. The ion-ion collision time in the bulk, τ_{iiB}, is

$$\tau_{iiB} = \tau_{iic} \frac{n_c}{n_B} \left[1 - \left(\frac{r}{R}\right)^p\right]^{3/2} \frac{\ln\Lambda_c}{\ln\Lambda_B} \quad ,$$

and using Eqs. (4) and (5), which give

$$\frac{n_c}{n_B} = \frac{4}{3} \left[1 - \left(\frac{r}{R}\right)^p\right]^{1/2} \left(\frac{R}{r_c}\right)^2 \left(\frac{r}{R}\right)^2 \quad ,$$

we have that

$$\tau_{iiB} \approx \frac{4}{3} \left(\frac{e\phi_{max}}{E_o}\right) \left(\frac{r}{R}\right)^2 \left[1 - \left(\frac{r}{R}\right)^p\right]^2 \left(\frac{\ln\Lambda_c}{\ln\Lambda_B}\right) \tau_{iic} \quad ,$$

where

$$\ln\Lambda_B \approx 7 + 2.3 \log_{10} \left[\frac{\left\{e\phi_{max} \left[1 - \left(\frac{r}{R}\right)^p\right]\right\}^{3/2}}{(n_B/10^{14}\ \mathrm{cm}^{-3})^{1/2}}\right] \quad .$$

For $r/R = 1/2$, $\ln\Lambda_B \approx 27$ and 25 for the EXP and REACTOR parameters, respectively.

The edge region is defined as that region in which the ion radial velocity slows down to a value of the order of it's birth speed, that is, $E_r = e\phi_{max} [1 - (r_e/R)^p] \sim E_o$. The radial extent of the edge region, $\Delta r_e = R - r_e$, is then given by $\Delta r_e \approx (1/p)(E_0/e\phi_{max})R$. The ion-ion collision time in the edge, τ_{iie}, is given by

$$\tau_{iie} = \tau_{iic} \left[\frac{E_0}{e\phi_{max}}\right]^{3/2} \frac{n_c}{n_{edge}} \frac{\ln\Lambda_c}{\ln\Lambda_e} \quad ,$$

and using Eq. (5), this becomes

$$\tau_{iie} \approx \left(\frac{E_0}{e\phi_{max}}\right) \left(\frac{\ln\Lambda_c}{\ln\Lambda_e}\right) \tau_{iic} \quad ,$$

where

$$\ln\Lambda_e \approx 7 + 2.3 \log_{10} \left[\frac{E_0^{3/2}}{(n_{edge}/10^{14}\ \mathrm{cm}^{-3})^{1/2}}\right] \quad ,$$

and $\ln\Lambda_e \approx 14$ and 7 for EXP and REACTOR parameters, respectively.

The collision time τ is not sufficient for comparing the effectiveness of collisions in the three spatial regions, because the time an ion spends in each region is drastically different. A better figure of merit is the ratio of the ion transit time t_{tr} through the region Δr to the collision time τ_{ii} in the region. This essentially gives the number of collisions, N_{coll}, that a test ion undergoes in a particular region of the device per transit.

The number of collisions in the core per pass is then, since $r_c \sim (E_0/e\phi_{max})^{1/2}R$,

$$(N_{coll})_c \sim \frac{2R}{\tau_{iic}} \left[\frac{E_o}{e\phi_{max}}\right]^{1/2} \left[\frac{m_i}{2e\phi_{max}}\right]^{1/2} \simeq 5 \times 10^{-8}, \text{ EXP}$$

$$\simeq 3 \times 10^{-4}, \text{ REACTOR} \quad .$$

Thus one collisional upscatter time corresponds to a large number of ion transits.

The ratio of the number of collisions in the edge region to the number of collisions in the core per transit is

$$\frac{(N_{coll})_{edge}}{(N_{coll})_c} = \frac{\Delta r_e}{r_c} \left[\frac{e\phi_{max}}{E_o}\right]^{1/2} \frac{\tau_{iic}}{\tau_{iie}} \quad ,$$

which is (take p = 3)

$$\frac{(N_{coll})_{edge}}{(N_{coll})_c} \sim \frac{1}{p} \left(\frac{e\phi_{max}}{E_o}\right) \left(\frac{\ln\Lambda_e}{\ln\Lambda_c}\right) \simeq 4 \times 10^2, \text{ EXP}$$

$$\simeq 2 \times 10^3, \text{ REACTOR} \quad ,$$

which corresponds to a small fraction of an edge collision per transit for experimental parameters, but to approximately one collision per transit for reactor parameters.

As representative values for the bulk region, we take r = R/2. Then the ratio of the number of collisions in the bulk region to the number of collisions in the core per pass is approximately

$$\frac{(N_{coll})_B}{(N_{coll})_c} \sim \frac{R/2}{r_c} \frac{\tau_{iic}}{\tau_{iiB}} \quad ,$$

which becomes

$$\frac{(N_{coll})_B}{(N_{coll})_c} \sim \frac{2R}{r_c} \left(\frac{E_o}{e\phi_{max}}\right)\left(\frac{\ln\Lambda_B}{\ln\Lambda_c}\right) \quad \cong 5 \times 10^{-2}, \text{ EXP}$$

$$\cong 2 \times 10^{-2}, \text{ REACTOR} \quad .$$

From the above, we see that the number of collisions per pass is by far the highest at the edge region, where the low energy ion distribution is relatively isotropic. The number of collisions is lowest in the bulk of the device, where the radial ion energy is high, and where the energy distribution is highly anisotropic, with $E_r >> E_{\perp}$.

III. HEURISTIC ESTIMATES OF ION "LOSS" RATES

In this section we give heuristic estimates of ion "loss" rates, in the sense of loss of ion convergence, in a SCIF device, due to the following ion-ion collisional effects: (1) upscatter in ion radial energy in the core region, and (2) perpendicular deflection in the bulk region.

The change in ion velocity by ion-ion scattering in a single transit through the bulk and core is expected to be small, since the ratio of transit time to collision time in the bulk and core of the device is small for both experimental and reactor parameters. For example, from Section II, the ratio of the transit time to the ion-ion collision time in the core is of the order of 5×10^{-8} for experimental parameters, and of the order of $\sim 3 \times 10^{-4}$ for reactor parameters. The ratio of the transit time to the ion-ion collision time in the bulk, for $r = R/2$, is of the order of $\sim 3 \times 10^{-9}$ for experimental parameters, and of the order of $\sim 6 \times 10^{-6}$ for reactor parameters. Therefore we treat upscatter as a diffusion process (in velocity space) in the core and bulk, rather than as a single scattering process. In analogy with the spatial diffusion problem, we use a continuity equation in velocity space as a basis for the loss process

$$\frac{dn}{dt} = D \frac{\partial^2 n}{\partial v^2} \quad ,$$

where the diffusion coefficient $D = (\delta v)^2/\tau$, with δv being the change in velocity produced in a time τ due to scattering in the dense core, and n is the time dependent density of ions in velocity space.

For the estimates in this section, we assume that the background ions are isotropic and are described by a Maxwellian distribution; we expect this to give a reasonable result at least to order of magnitude. The diffusion tensor in the Fokker-Planck equation, $D_{ij} = \langle \Delta v_i \Delta v_j \rangle / 2\tau$, is diagonal in this case, and can be expressed in terms of components parallel, $D_\parallel$, and perpendicular, $D_\perp$, to the direction of the test particle initial velocity (before the collision). These components are[5]

$$D_\parallel = \frac{e^4 n' \ln\Lambda'}{4\pi\epsilon_0^2 m_i^2 v} \frac{\Phi_1(v/v')}{2(v/v')^2} \quad ,$$

$$D_\perp = \frac{e^4 n' \ln\Lambda'}{8\pi\epsilon_0^2 m_i^2 v} \left[\Phi\,(v/v') - \frac{\Phi_1(v/v')}{2(v/v')^2}\right] \quad , \tag{7}$$

where the unprimed, primed quantities refer to the test, field ions, respectively; the temperature T' for the field ions is $(3/2)\,T' = (1/2)\,m_i v'^2$, $\ln\Lambda'$ is the Coulomb logarithm for the field ions, and $\Phi_1(x)$ is defined in terms of the error function

$$\Phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp\,(-\zeta^2)\,d\zeta \quad ,$$

by

$$\Phi_1(x) = \Phi(x) - \frac{2x}{\sqrt{\pi}} \exp(-x^2) = \Phi(x) - x\,\frac{\partial\Phi}{\partial x} \quad .$$

For $x = 1$, $\Phi_1/2x^2 = 0.214$, and $\Phi = 0.843$.[6] For $x > 2$, $\Phi_1 \approx \Phi \approx 1$.[5]

A. ION LOSS DUE TO RADIAL UPSCATTER IN THE CORE

Upscatter in radial velocity in the core by collisions can affect ion convergence in the device. Consider a collision which causes one ion to gain energy and another ion to lose energy. The ion which gains energy in the core can make a larger radial excursion to a radius beyond it's birth radius, with the possibility that the magnetic fields at the new turning point will be sufficient to deflect the ion in $v_\perp$, leading to a degradation of the focus.

We describe radial upscatter in the core by a diffusion equation

$$\frac{dn}{dt} = D_{\|eff} \frac{\partial^2 n}{\partial v_\|^2} \quad .$$

Here $D_{\|eff} = D_{\|c}\,(t_c/t_D)$, where $D_{\|c}$ is the parallel diffusion coefficient in the core, given by Eq. (7) with $1/2\, m_i v'^2 \approx e\phi_{max}$ and $n' = n_c$, and (t_c/t_D) is the fractional time that a test ion spends in the core per transit through the device. Assuming the core has radius r_c, with uniform density n_c, the transit time through the core is given approximately by $t_c = 2r_c/v_c$, where v_c is the velocity of the test ion in the core region. The time for a test ion to transit through the entire device is given approximately by $t_D = 2r_o/\bar{v}_r$, where r_o is the radial birth location of the test ion, and $\bar{v}_r$ is the average velocity of the test ion in the device, given by $\bar{v}_r \simeq (1/R) \int (2e\phi_{max}/m_i)^{1/2} (1 - (r/R)^p)^{1/2}\, dr = \pi/4\, (2e\phi_{max}/m_i)^{1/2} = \pi/4\, v_c$, for $p = 2$. Thus

$$D_{\|eff} \approx \frac{1}{4} \left[\Phi_1 \left(\frac{v}{v'}\right)\right] \left(\frac{v'}{v}\right)^3 v'^2 \left(\frac{r_c}{r_o}\right) \left(\frac{\bar{v}_r}{v_c}\right) \frac{1}{\tau_{\|c}} \quad ,$$

where

$$\tau_{\parallel c} = 2\pi\epsilon_o^2 m_i^2 v'^3/(e^4 n_c \ln\Lambda_c) \quad .$$

Since the test particle is actually one of the core particles, we assume that $v = v'(1 + f)$, where $f = \Delta v/v'$ and Δv is the upscatter in the test particle velocity due to collisions in the core. Then

$$D_{\parallel eff} \approx [\Phi_1(1 + f)] \frac{1}{(1 + f)^3} v'^2$$

$$\cdot \left(\frac{r_c}{r_o}\right)\left(\frac{\bar{v}_r}{v}\right) \frac{n_2(10^{12}\text{cm}^{-3})}{\phi^{3/2}\ (10\ \text{kV})} \quad .$$

We estimate the ion "loss" rate τ_{loss} due to upscatter in radial velocity using

$$\tau_{loss} = \frac{(\Delta v_r)^2}{D_{\parallel eff}} \quad ,$$

where $\Delta v_r \approx f v_r'$ is the increase in the test ion radial velocity due to collisions in the core, which we relate to the loss of focus below. The loss time due to upscatter would then be given by

$$\tau_{loss} \approx \frac{v'^2 f^2}{D_{\parallel eff}}$$

$$\approx f^2(1 + f)^3 \frac{1}{[\Phi_1(1 + f)]} \left(\frac{r_o}{r_c}\right)\left(\frac{v_c}{\bar{v}_r}\right) \frac{\phi^{3/2}\ (10\ kV)}{n_2(10^{12}\ cm^{-3})}\ s \quad . \quad (8)$$

We estimate the value of f which could lead to ion "loss," in the sense of a broadened core radius r_c due to increased deflection as the upscattered ion makes a larger radial excursion to a higher B-field region. To do this, we estimate the new radial location $r_o' > r_o$ at which the upscattered ion's radial energy decreases to its birth energy. Neglecting scattering, we assume the ion radial energy profile is, generalizing Eq. (3) to $r_o \neq R$,

$$E_r = E_o + e\phi_{max} \left[\left(\frac{r_o}{R}\right)^p - \left(\frac{r}{R}\right)^p\right] \quad ,$$

where typically p is related to the magnetic-field profile by p ~ m. Due to upscatter in the core, the ion receives a kick in energy ΔE_r,

$$\Delta E_r \sim e\phi_{max} \left(\frac{r_o}{R}\right)^p (2 + f)f \quad .$$

Then $E_r + \Delta E_r = E_o$ when

$$\frac{r_o'}{R} \sim (1 + f)^{2/p} \left(\frac{r_o}{R}\right) \quad .$$

Since the perpendicular deflection of the ion by the magnetic field, ΔV, at the ion birth location is related to the core radius by[2]

$$\frac{r_c}{R} = \frac{\Delta V}{V} \quad ,$$

the increase in core radius, Δr_c, due to larger deflection at r_o' is then

$$\frac{\Delta r_c}{r_c} = \frac{\Delta V(r_o') - \Delta V(r_o)}{\Delta V(r_o)} \quad .$$

The perpendicular deflection of the ion by the magnetic field in the SCIF device was estimated in Ref. 2 by considering ion orbits in orthonormal magnetic and electric fields:

$$\frac{\Delta V(r_o)}{V} \simeq \frac{\omega_{ci}^2}{(m + 1)^2} \left(\frac{r_o}{R_B}\right)^{2m} \frac{r_o^2}{[2e\phi_o/m_i]} \quad .$$

where ϕ_o is the potential drop from $r = 0$ to $r = r_o$. At r_o',

$$\frac{\Delta V(r_o')}{V} \simeq \frac{\omega_{ci}^2}{(m + 1)^2} \frac{1}{R_B^{2m}} \frac{(1 + f)^{2(2m+2/p)} r_o^{2m+2}}{[2e\phi_o/m_i]} \quad ,$$

so that the change in ΔV due to a larger radial excursion to r_o' is, for $f < 1$,

$$\frac{\Delta V(r_o') - \Delta V(r_o)}{\Delta V(r_o)} = \frac{\Delta r_c}{r_c} = \frac{2(2m + 2)f}{p} \quad .$$

Thus $\Delta r_c/r_c > 1$ requires f comparable to unity. We estimate the ion "loss" time from Eq. (8) evaluated with $f \sim 1$. For $r_0 = R$, and for the EXP parameters given in Table I,

$$\tau_{loss} \sim 450 \text{ s} \quad .$$

For REACTOR parameters,

$$\tau_{loss} \sim 45 \text{ ms} \quad ,$$

as compared with discharge time scales of ~ 10 ms.

B. ION LOSS DUE TO PERPENDICULAR DEFLECTION IN THE MANTLE

Perpendicular deflection due to scattering in the bulk of the device could lead to isotropization of the anisotropic ion distribution. From conservation of angular momentum (without scattering), $rv_\perp(r)$ = constant. An increase in the azimuthal ion velocity, $\Delta v_\perp$, can be related to an increase in the core convergence radius, Δr_c, by[2]

$$\frac{\Delta v_\perp(r)}{v_\perp(r)} = \frac{\Delta r_c}{r_c} \quad .$$

Thus isotropization in the bulk region, resulting in a transfer of $v_r \rightarrow v_\perp$ in the bulk, could degrade the ion focus. In this section, we estimate the rate at which ions are "lost" to the dense core in the sense that they converge to a significantly larger core radius.

We use a continuity equation in velocity space as the basis for the loss process

$$\frac{\partial n}{\partial t} = D_{\perp eff} \frac{\partial^2 n}{\partial v_\perp^2} \quad ,$$

where $D_{\perp eff}$ is the effective perpendicular diffusion coefficient in the mantle

$$D_{\perp eff} = D_\perp \, (t_B/t_D) \quad ,$$

with $D_\perp$ the perpendicular diffusion coefficient in the bulk, given by Eq. (7) with $T' \sim e\phi_{max}$ and $n' = n_B$, and (t_B/t_D) is the fractional time that a test ion spends in the bulk per transit through the device.

As representative values for calculating the loss rate due to deflection in the bulk of the device, we use $r = R/2$, and we assume the background ions in the bulk region have a uniform density n_B $(r = R/2)$ given by (see Eq. (4))

$$n_B \, (r = R/2) = 3n_{edge} \left(\frac{E_0}{e\phi_{max}}\right)^{1/2} \quad ,$$

and a radial energy given by $E_r \sim e\phi_{max}$. The transit time for a test ion in the bulk region is then of order of 1/2 its transit time through the entire device. The effective perpendicular diffusion coefficient is then

$$D_{\perp eff} \sim \frac{1}{8} \frac{1}{\tau_{\parallel B}} v'^2 \left(\frac{v'}{v}\right) \left[\Phi \left(\frac{v}{v'}\right) - \frac{\Phi_1(v/v')}{2(v/v')^2}\right] \quad ,$$

where $\tau_{\|B} \sim \tau_{\|c}\ (n_c/n_B)(\ln\Lambda_c/\ln\Lambda_B) \sim 1/3\ (e\phi_{max}/E_0)\ \tau_{\|c}$, and $v' \approx v'_r \approx (2e\phi_{max}/m_i)^{1/2}$.

We estimate the ion "loss" time, in the sense of ion focus degradation, as the time for $(\Delta v_\perp/v_\perp)$, and thus $(\Delta r_c/r_c)$, to become of order q, where q is a number comparable to or greater than 1. By conservation of angular momentum, $v_\perp = v_{\perp 0}\ (R/r)$ in the bulk, so that $\Delta v_\perp = qv_\perp = 2qv_{\perp 0}$ at $r = R/2$. The ion loss time can then be estimated from

$$\tau_{loss} \sim \frac{(\Delta v_\perp)^2}{D_{\perp eff}} \sim 11\ q^2\ \tau_{\|c}\ \left(\frac{v}{v'}\right) \frac{1}{\left[\Phi\ \left(\frac{v}{v'}\right) - \frac{\Phi_1(v/v')}{2(v/v')^2}\right]} ,$$

$$\sim 18\ q^2\ \tau_{\|c} \qquad \text{for } v = v' , \tag{9}$$

with

$$\tau_{\|c} \sim 0.3 \text{ sec} \cdot \frac{[e\phi_{max}(10 \text{ keV})]^{3/2}}{n_c(10^{12}\text{cm}^{-3})} .$$

For the experimental parameters in Table I, the loss time is of order 5 q^2 sec. For reactor parameters, the ion loss time due to this collisional process is of the order of

$$\tau_{loss} \sim 0.2\ q^2 \text{ ms} ,$$

as compared with discharge time scales of ~ 10 ms.

Note that the loss time is proportional to the $(\Delta v_\perp)^2$ required for loss, which is proportional to the $(\Delta r_c)^2$ required for loss. The value of $\Delta v_\perp$ required for loss will be calculated later in this report.

C. THERMALIZATION BY COLLISIONS AT THE EDGE

A test ion spends approximately the same amount of time transiting the core region as it does transiting the thinner edge region. That is, the transit time in the core, t_c, where

$$t_c \sim 2r_c/v_c \sim 2R \left(\frac{E_o}{e\phi_{max}}\right)^{1/2} \left(\frac{m_i}{2e\phi_{max}}\right)^{1/2} \quad ,$$

is comparable to the transit time in the edge region, t_{edge}, where

$$t_{edge} \sim \frac{2\Delta r_e}{v_o} \sim \frac{2R}{p} \left(\frac{E_o}{e\phi_{max}}\right) \left(\frac{m_i}{2E_o}\right)^{1/2} \sim \frac{1}{p}\, t_c \quad .$$

From Section II, the ion-ion collision frequency is larger at the edge than in the core, viz.,

$$\frac{\tau_{iie}}{\tau_{iic}} \sim \left(\frac{E_o}{e\phi_{max}}\right) \quad .$$

We estimate whether the ion distribution might thermalize in the edge region. The thermalization time scale is of the order of the ion-ion self collision time τ_{iie}. The amount of time that a test ion spends in the edge region is t_{edge} times the number of passes through the device.

For experimental parameters, $\tau_{iie} \sim 1$ sec $\cdot$ $(E_0/e\phi_{max}) \sim 5 \times 10^{-4}$ sec. The ion transit time through the edge region would be of the order of 10 ns for $R = 100$ cm. Assuming 10^4 transits through the device, a test ion would spend about 10^{-4} sec in the edge region, somewhat less than the ion-ion collision time.

For reactor parameters, however, $\tau_{iie} \sim (3 \times 10^{-5}$ sec$) \cdot (E_0/e\phi_{max}) \sim 1.5$ ns. The ion transit time through the edge region would also be of the order of 1.5 ns for $R = 200$ cm, $p = 2$. Thus in several passes through the device, the ion distribution at the edge could be thermalized.

IV. CALCULATION OF THE ION DISTRIBUTION PRODUCED BY COLLISIONS

In Section II we calculated the relative rates of collisions in the various spatial regions of a SCIF, and found that collisions in the edge regions dominate. More importantly, we found that the collision rate in the interior of the device is very small compared with the transit frequency, so that for many transits the ions return to their original turning points at $r = R$. That is, the definition of "edge region" is preserved for thousands of interior plasma collision times. In Section III we estimated collisional ion "loss" time scales, and found that these time scales are much longer than discharge time scales for experimental parameters, but not for reactor parameters. We also found, however, that collisions in the edge region might relax the anisotropy in that region. In this section we give a Fokker-Planck-like calculation of temperature relaxation of the edge distribution due to ion-ion collisions throughout the device. Since the collision frequencies in the bulk and core are much lower than the transit frequency, the edge distribution determines the evolution of the ion density and energy structure in all three regions. The edge distribution evolves (a) due to collisions at the edge, (b) due to the influx of energy-scattered ions (due to collisions in the core) and (c) due to energy-deflected ions (due to collisions in the bulk) coming into the edge region in every transit.

The dominant effect of edge collisions is to relax any temperature anisotropy which develops in this region. The source of ion anisotropy in the edge is the influx of excess $v_\perp$ into the edge region due to the conversion of radial to azimuthal energy via ion-ion collisions in the highly anisotropic bulk plasma. The goal of the present calculation is to quantify the extent to which isotropizing collisions in the low ion energy edge can remove or reduce this increase of $v_\perp$. The anisotropy at the edge is the primary effect which spreads out the distance of closest approach to $r = 0$ (this is obvious from energy conservation), which in turn could lead to an isotropic interior.

A. ION DISTRIBUTIONS IN THE THREE REGIONS

In Sections IV.A and IV.B, we assume that the effect of the competition between edge collisions and bulk collisions will be that the edge region will ultimately tend toward a bi-Maxwellian, characterized by T_r and $T_\perp$, and calculate the steady state difference of T_r and $T_\perp$ in the edge region.

Thus for the edge region, we take

$$f_{i,e} = K_e e^{-\frac{m_i v_r^2}{2T_{ro}}} e^{-\frac{m_i v_\perp^2}{2T_{\perp o}}} \quad , \tag{10}$$

with K_e being a normalization constant such that

$$n_{edge} = \int d^3 \underset{\sim}{v}\, f_{i,e} \quad .$$

Since the ion collision frequency in the interior is low, the ions maintain a dominant set of turning points at $r = R$, so that the interior distribution continues to be the evolution of an edge Maxwellian under the influence of a strong central potential $\phi(r)$. Therefore, we take the ion distribution in the bulk region to be the sum of two radially counter-drifting bi-Maxwellians. The radial drift, $v_D = \sqrt{2e\phi(r)/m_i}$, is due to the electrostatic field. The azimuthal temperature $T_\perp$ increases with decreasing radius, reflecting conservation of angular momentum, $T_\perp(r) = T_{\perp o}\,(R/r)^2$. Thus in the bulk

$$f_{i,B} = K_B e^{-\frac{m_i(v_r \pm v_D)^2}{2T_{ro}}} \cdot e^{-\frac{m_i v_\perp^2}{2T_\perp(r)}} , \tag{11}$$

where K_B is a normalization constant such that

$$n_B = \int d^3 \underset{\sim}{v}\, f_{i,B} \quad .$$

We note that the bulk distribution reduces to the edge distribution at $r = R$, since $\phi(R) = 0$.

We assume that the core is defined as that region in which the azimuthal and radial ion energies are the same, that is, $T_r = T_\perp = e\phi_{max}$. Thus we model the core distribution as an isotropic Maxwellian at the temperature $T_c = e\phi_{max}$, since ϕ is virtually constant over the small core region,

$$f_{i,c} = K_c e^{-\frac{m_i v^2}{2T_c}} \quad . \tag{12}$$

For simplicity, we use the density and energy profiles in the three regions of the device derived in Section II, which can be shown to be similar to those produced by a bi-Maxwellian.

B. CALCULATION OF THE STEADY-STATE ION DISTRIBUTION

We model the evolution of $T_\perp$ in the edge region by:

$$\frac{dT_{\perp o}}{dt} = \left.\frac{dT_{\perp o}}{dt}\right|_B + \left.\frac{dT_{\perp o}}{dt}\right|_{ISO} \quad . \tag{13}$$

Here, $dT_{\perp o}/dt|_B$ is the source term for the additional $v_\perp$ coming in to the edge region on each ion transit, due to the conversion of radial to azimuthal energy by collisions in the bulk, and $dT_{\perp o}/dt|_{ISO}$ is the isotropization rate due to collisions in the edge region. There is a similar equation for T_{ro}. However, since $(mv_r^2/2) = T_r/2$, $mv_\perp^2/2 = T_\perp$, we can use conservation of energy to write[7]

$$\frac{dT_{\perp o}}{dt} = - \frac{d}{dt}\frac{T_{ro}}{2} \quad , \tag{14}$$

and to obtain an equation for the evolution of the edge temperature difference, $\Delta T = T_{\perp o} - T_{ro}$,

$$\frac{d}{dt}\Delta T = 3\left.\frac{dT_{\perp o}}{dt}\right|_B + 3\left.\frac{dT_{\perp o}}{dt}\right|_{ISO} \quad . \tag{15}$$

There is a similar driver in which T_{ro} is changed by energy scattering in the core, with $T_{\perp o}$ in the edge changed because energy conservation slightly alters the turning point. We emphasize here the $T_\perp$ scattering because $T_\perp$ acquired during a transit is carried out to the turning point decreased only by r/R (of order 1/2), while v_r <u>always</u> goes to 0 at the turning point because of the structure of $\phi(r)$.

r/R (of order 1/2), while v_r <u>always</u> goes to 0 at the turning point because of the structure of $\phi(r)$.

In the following calculation, we assume that the edge temperature difference is small, $\Delta T = T_{\perp o} - T_{ro} \ll T$, because the conversion of radial to azimuthal velocity of an ion in each transit through the bulk is small compared with the rate of isotropizing edge collisions, from the estimates in Section II.

The isotropization time for a bi-Maxwellian distribution, in the limit of small temperature differences, is given by:[7]

$$\left.\frac{dT_{\perp o}}{dt}\right|_{ISO} = -\frac{\Delta T}{\tau_e} \quad , \tag{16}$$

where $\tau_e = (15/4)\,(m_i^{1/2}\,T_{edge}^{3/2}/\sqrt{\pi}\,e^4\,n_{edge}\,\ln\Lambda_e)$, and $T_{edge} = T_{ro}$ to this order of approximation.

To calculate $(dT_{\perp o}/dt)$ due to the bulk region, where the ion energy distribution is highly anisotropic, we write the Fokker-Planck collision term using the Landau form for the collision operator. We take the energy moment of the kinetic equation

$$\frac{\partial f_i}{\partial t} = -\frac{\partial j_\alpha}{\partial v_\alpha} \quad , \tag{17}$$

where j is the three-dimensional particle flux density in velocity space, representing particle collisions:

$$j_\alpha = a_\alpha f_i - d_{\alpha\beta}\,\frac{\partial f_i}{\partial v_\beta} \quad , \tag{18}$$

where

$$a_\alpha = \frac{1}{\tau}\left[\langle\Delta v_\alpha\rangle - \frac{1}{2}\frac{\partial}{\partial v_\beta}\langle\Delta v_\alpha \Delta v_\beta\rangle\right] ,$$

$$d_{\alpha\beta} = \frac{1}{2\tau}\langle\Delta v_\alpha \Delta v_\beta\rangle ,$$

as given for example in Reference 8.

Using conservation of energy, we obtain for the increase of $T_{\perp o}$ due to the bulk,[9]

$$\frac{dT_{\perp o}}{dt} = -\frac{d}{dt}\frac{T_{ro}}{2} = -\frac{1}{n_B}\int \frac{m_i v_r^2}{2}\frac{\partial f_i}{\partial t}\, d^3\underset{\sim}{v} = -\frac{m_i}{n_B}\int v_r j_r d^3\underset{\sim}{v} \quad . \tag{19}$$

We compute j_r by writing the flux in this case in *Landau form*[9]

$$j_r = \frac{\ln\Lambda}{8\pi}\left(\frac{4\pi e^2}{m_i}\right)^2 \int U_{rj}\left(f_i \frac{\partial f_i'}{\partial v_j'} - f_i' \frac{\partial f_i}{\partial v_j}\right) d^3\underset{\sim}{v}' \quad , \tag{20}$$

with

$$U_{rj} = \frac{\delta_{rj}}{u} - \frac{u_r u_j}{u^3} \quad , \quad \text{so that} \quad U_{rr} = \frac{u_\perp^2}{u^3} \quad ,$$

and $$U_{r\perp} = -\frac{u_r u_\perp}{u^3} \quad .$$

Here the primed, unprimed quantities refer to the field, test particles, respectively, and $u = |\underline{v} - \underline{v}'|$. We assume that the field and test particles counter-drift radially:

$$f_i = K_B e^{-\frac{m_i(v_r - v_D)^2}{2T_{ro}}} e^{-\frac{m_i v_\perp^2}{2T_\perp}} \quad , \tag{21}$$

$$f_i' = K_B e^{-\frac{m_i(v_r' + v_D)^2}{2T_{ro}}} e^{-\frac{m_i v_\perp'^2}{2T_\perp}} \quad . \tag{22}$$

Then

$$\frac{\partial f_i}{\partial v_r} = - \frac{m_i}{T_{ro}} (v_r - v_D) f_i \quad , \quad \frac{\partial f_i}{\partial v_\perp} = - \frac{m_i v_\perp}{T_\perp} f_i$$

$$\frac{\partial f_i'}{\partial v_r'} = - \frac{m_i}{T_{ro}} (v_r' + v_D) f_i' \quad , \quad \frac{\partial f_i'}{\partial v_\perp'} = - \frac{m_i v_\perp'}{T_\perp} f_i' \quad .$$

Then

$$j_r = - \frac{\ln\Lambda}{8\pi} \left(\frac{4\pi e^2}{m_i}\right)^2 \int \Big[U_{rr} f_i f_i' \left(\frac{m_i}{T_{ro}}\right)(- u_r + 2v_D)$$

$$+ U_{r\perp} f_i f_i' \left(\frac{m_i}{T_\perp}\right)(- u_\perp) \Big] \quad ,$$

which can be written as

$$j_r = \frac{\ln\Lambda}{8\pi} \left(\frac{4\pi e^2}{m_i}\right)^2 \int f_i f_i' \left[\frac{m_i}{T_{ro}} \frac{u_\perp^2}{u^3} (u_r - 2v_D) - \frac{m_i}{T_\perp} \frac{u_r u_\perp^2}{u^3}\right] ,$$

and, using

$$\int d^3\underline{v} = 2\pi \int_{-\infty}^{\infty} dv_r \int_0^{\infty} \frac{dv_\perp^2}{2} ,$$

Eq. (19) becomes

$$\frac{dT_{\perp o}}{dt} = - A \int_{-\infty}^{\infty} dv_r dv_r' \int_0^{\infty} dv_\perp^2\, dv_\perp'^2\, v_r$$

$$\cdot \left[\frac{u_\perp^2}{u^3} \frac{m_i}{T_{ro}} (u_r - 2v_D) - \frac{u_r u_\perp^2}{u^3} \frac{m_i}{T_\perp}\right] f_i f_i' ,$$

where A is given by

$$A = \frac{m_i}{n_B} \pi^2 \frac{\ln\Lambda}{8\pi} \left(\frac{4\pi e^2}{m_i}\right)^2 .$$

To do this integral, we make the approximation that $u = |\underline{u}| \sim |v_r - v_r'| \sim 2\, v_D$, so that u can be taken out of the integral. We rewrite the integration variables as

$$\bar{v}_r = v_r - v_D \qquad ,$$

$$\bar{v}_r' = v_r' + v_D \qquad ,$$

$$u_r = v_r - v_r' = \bar{v}_r - \bar{v}_r' + 2v_D$$

to obtain

$$\frac{dT_{\perp o}}{dt} = - AK_B^2 \int \frac{(\bar{v}_r + v_D)}{(2v_D)^3} \left[\frac{m_i u_\perp^2}{T_{ro}} (u_r - 2v_D) - \frac{m_i u_\perp^2}{T_\perp} u_r\right]$$

$$\cdot\, e^{-\frac{m_i \bar{v}_r^2}{2T_{ro}}} e^{-\frac{m_i \bar{v}_r'^2}{2T_{ro}}} e^{-\frac{m_i v_\perp^2}{2T_\perp}} e^{-\frac{m_i v_\perp'^2}{2T_\perp}} d\bar{v}_r\, d\bar{v}_r'\, dv_\perp^2\, dv_\perp'^2, \quad (23)$$

which can be rewritten as

$$\frac{dT_{\perp o}}{dt} = - \frac{AK_B^2}{(2v_D)^3} \int (v_r + v_D)\left[\frac{m_i u_\perp^2}{T_{ro}} (\bar{v}_r - \bar{v}_r') - \frac{m_i u_\perp^2}{T_\perp} (\bar{v}_r - \bar{v}_r' + 2v_D)\right]$$

$$\cdot\, e^{-\frac{m_i \bar{v}_r^2}{2T_{ro}}} e^{-\frac{m_i \bar{v}_r'^2}{2T_{ro}}} e^{-\frac{mv_\perp^2}{2T_\perp}} e^{-\frac{mv_\perp'^2}{2T_\perp}} d\bar{v}_r \, d\bar{v}_r' \, dv_\perp^2 \, dv_\perp'^2 \quad .$$

In the integrations over $d\bar{v}_r$, $d\bar{v}_r'$, only those terms which are proportional to even powers of $\bar{v}_r$, $\bar{v}_r'$, are non-zero; thus the integral over $d\bar{v}_r$, $d\bar{v}_r'$ is

$$I_1 = \int_{-\infty}^{\infty} d\bar{v}_r \int_{-\infty}^{\infty} d\bar{v}_r' \left[\frac{m_i u_\perp^2}{T_{ro}} \bar{v}_r^2 - \frac{m_i u_\perp^2}{T_\perp} (\bar{v}_r^2 + 2v_D^2)\right]$$

$$\cdot\, e^{-\frac{m_i \bar{v}_r^2}{2T_{ro}}} e^{-\frac{m_i \bar{v}_r'^2}{2T_{ro}}} \quad ,$$

which is

$$I_1 = \frac{\pi}{2} \left(\frac{2T_{ro}}{m_i}\right)^2 m_i u_\perp^2 \left(\frac{1}{T_{ro}} - \frac{1}{T_{\perp o}}\right) - \pi \left(\frac{2T_{ro}}{m_i}\right) 2v_D^2 \frac{m_i u_\perp^2}{T_\perp}$$

$$= 2\pi \frac{T_{ro}}{m_i} u_\perp^2 \left(1 - \frac{T_{ro}}{T_\perp} - \frac{2m_i v_D^2}{T_\perp}\right) \quad ,$$

so that

$$\frac{dT_{\perp o}}{dt} = - \frac{AK_B^2}{(2v_D)^3} \int_0^\infty u_\perp^2 \left(\frac{2\pi T_{ro}}{m_i}\right) \left(1 - \frac{T_{ro}}{T_\perp} - \frac{2m_i v_D^2}{T_\perp}\right)$$

$$\cdot\, e^{-\frac{m_i v_\perp^2}{2T_\perp}} e^{-\frac{m_i v_\perp'^2}{2T_\perp}} dv_\perp^2 \, dv_\perp'^2 \quad .$$

To do the $v_\perp$, $v_\perp'$ integrations, we write $u_\perp^2 = (v_\perp - v_\perp')^2$, so that the integration over $dv_\perp$, $dv_\perp'$ becomes

$$I_2 = \int_0^\infty dv_\perp^2 \int_0^\infty dv_\perp'^2 \, (v_\perp^2 - 2v_\perp v_\perp' + v_\perp'^2) \, e^{-\frac{m_i v_\perp^2}{2T_\perp}} e^{-\frac{m_i v_\perp'^2}{2T_\perp}}$$

$$= \left(\frac{2T_\perp}{m_i}\right)^3 [2\Gamma(2)\Gamma(1) - 2\Gamma(1.5)\Gamma(1.5)]$$

$$= \left(\frac{2T_\perp}{m_i}\right)^3 \left(2 - \frac{\pi}{2}\right) \quad ,$$

and we have

$$\frac{dT_{\perp o}}{dt} = - \frac{AK_B^2}{(2v_D)^3} \left(\frac{2\pi T_{ro}}{m_i}\right) \left(\frac{2T_\perp}{m_i}\right)^3 \left[1 - \frac{T_{ro}}{T_\perp} - \frac{2m_i v_D^2}{T_\perp}\right] \left(2 - \frac{\pi}{2}\right) \quad . \quad (24)$$

Then with the normalization constant K_B given by

$$K_B = n_B \left(\frac{m_i}{2\pi T_\perp}\right) \sqrt{\frac{m_i}{2\pi T_{ro}}} \qquad ,$$

we have that

$$\frac{AK_B^2}{(2v_D)^3} \left(\frac{2\pi T_{ro}}{m_i}\right) \left(\frac{2T_\perp}{m_i}\right)^3 \left(2 - \frac{\pi}{2}\right) = \frac{n_B}{m_i^2} \frac{2\pi T_\perp}{(2v_D)^3} \ln\Lambda \; e^4 \; (4 - \pi) \qquad .$$

We rewrite Eq. (24) as

$$\frac{dT_{\perp o}}{dt} = \frac{(2m_i v_D^2 + T_{ro} - T_\perp)}{\tau_B} \qquad , \tag{25}$$

where the isotropization time τ_B is given by

$$\tau_B = \frac{m_i^2 \; (2v_D)^3}{2\pi e^4 \; (\ln\Lambda_B) \; n_B \; (4 - \pi)} \qquad . \tag{26}$$

In order to use this as a source term for $(dT/dt)_B$ in the edge region, it is necessary to take into account the fact that an ion in it's transit through the device spends more time in the bulk than in the edge. Thus, we multiply Eq. (25) by the ratio of the transit time in the bulk to the transit time at the edge (t_B/t_e). This is equivalent to writing Eq. (25) in units of the edge transit time. We also need to take the following effect into account, which is a consequence of conservation of angular momentum. An ion which gains excess $(1/2)m_i v_\perp^2$ in the bulk at radius r, will bring an excess azimuthal energy into the edge of magnitude $\approx (1/2)m_i v_\perp^2 (r/R)^2$. We thus have, for use in Eq. (13),

$$\left.\frac{dT_{\perp o}}{dt}\right|_B \approx \frac{2m_i v_D^2}{\tau_B} \left(\frac{r}{R}\right)^2 \frac{t_B}{t_e} \quad . \tag{27}$$

Equation (15) then becomes

$$\frac{d}{dt}\,\Delta T \approx \frac{6m_i v_D^2}{\tau_B} \left(\frac{r}{R}\right)^2 \frac{t_B}{t_e} - \frac{3\Delta T}{\tau_e} \quad . \tag{28}$$

We use the estimates for the transit time in the bulk and the edge regions from Section II. The ratio of the transit time in the bulk to the transit time in the edge is of the order of $t_B/t_e \sim [R/(E_o/e\phi_{max})(R/p)]\ [(E_o^{1/2})/(e\phi_{max})^{1/2}] \sim p\sqrt{e\phi_{max}/T_{ro}}$, while the ratio of the isotropization time in the bulk, τ_B, to the isotropization time in the edge, τ_e, is

$$\frac{\tau_B}{\tau_e} \sim \left[\frac{8m_i^{1/2}}{2\pi e^4 n_B} \frac{(e\phi_{max})^{3/2}}{(4-\pi)\ln\Lambda_B}\right]\left[\frac{4\sqrt{\pi}\ e^4\ n_{edge}\ \ln\Lambda_e}{15\ m_i^{1/2}\ E_o^{3/2}}\right] \quad ,$$

which becomes

$$\tau_B/\tau_e \sim \left(\frac{e\phi_{max}}{T_{ro}}\right)^{3/2} \frac{n_{edge}}{n_B} \quad . \tag{29}$$

Thus, as long as $n_{edge}/n_B > p$, with p defined in Eq. (2) typically of order 2-3, we expect that the temperature difference in the edge region would remain small, $\Delta T < T_{ro}$. This condition depends on the form of the density profile in the device, which is determined from the edge ion distribution and conservation of energy and angular momentum. Using the flattop ion distribution discussed in Section II, we have that $n_{edge}/n_B \sim (e\phi_{max}/E_o)^{1/2} \gg 1$.

The equilibrium solution of Eq. (28) yields $\Delta T = (T_\perp - T_r)_{edge} \sim T_{ro} (T_{ro}/e\phi_{max})^{1/2} \ll T_{ro}$. Thus the quasisteady solution to this nonlocal collision relaxation process is an ion distribution which is nearly Maxwellian at the edge. Since the ions have turning points in the same vicinity as their birth point, the interior distribution continues to evolve from the edge distribution, conserving energy and angular momentum, so that the non-Maxwellian distributions Eqs. (21)-(22) with $T_\perp = T_{\perp o} (R/r)^2$ continue to describe the bulk plasma. The interior, like the edge, is driven toward isotropy by local collisions, but driven toward anisotropy by influx of ions from the Maxwellian edge. It is because the edge collision frequency per transit is many orders of magnitude larger than the bulk rate that the anisotropic influx so dominates the interior.

V. APPLICATION TO SCIF ION LOSS RATES

For the experimental parameters in Table I, the rate of collisions per transit is so small as to be negligible. For reactor parameters, however, the collision rates must be compared to the fusion energy multiplication rate, since about 10^4 transits before loss are required for economic operation. Therefore in this section we consider in detail the loss produced by collisions for reactor parameters in a magnetically confined SCIF.

The previous results show that edge collisions maintain a non-Maxwellian interior for a time larger than the small angle scattering time which in most cases produces a local Maxwellian. Applying this result to SCIF loss rates, we note that there are two effects. First, large angle scattering will prevent ions from returning to the vicinity of their birth location, thus removing the healing effect of edge collisions which assumes a fixed range of turning points. Second, the slightly anisotropic edge distribution found in Section III will produce some degradation in the core on the time scale of the small angle scattering time in the core.

A. ION LOSS BY LARGE ANGLE SCATTERING

The frequency of 90° scattering is obtained by simply setting $\log \Lambda = 1$ in the collision rates in Section III. To estimate the loss rate we must estimate how large a change in velocity Δv is required to cause an ion to be lost to the core. In this section we consider in detail such losses for reactor parameters in a magnetically confined SCIF.

1. Ion Loss Due to Radial Upscatter in the Core

Upscatter in radial velocity in the core by collisions can affect ion convergence in the device. Consider a collision which causes one ion to gain energy and another ion to lose energy. The ion which gains energy in the core can make a larger radial excursion to a radius beyond it's birth radius, with the possibility that the magnetic fields at the new turning point will be sufficient to deflect the ion in $v_\perp$, leading to a degradation of the focus.

We estimate the value of δv which could lead to ion "loss," in the sense of a broadened core radius r_c, due to increased deflection by the higher B-field as the upscattered ion makes a larger radial excursion. To do this, we estimate the new radial location r_o' at which the upscattered ion's radial energy decreases to its birth energy. Neglecting scattering, we assume the ion radial energy profile is, generalizing Eq. (3) to $r_o \neq R$,

$$E_r = E_o + e\phi_{max}\left[\left(\frac{r_o}{R}\right)^p - \left(\frac{r}{R}\right)^p\right] \quad , \tag{30}$$

Due to upscatter in the core, the ion receives a kick in energy ΔE_r,

$$\Delta E_r = e\phi_{max}\left(\frac{r_o}{R}\right)^p f(2 + f) \quad . \tag{31}$$

where $f = \delta v_r/v_r$.

Then $E_r + \Delta E_r = E_o$ when

$$E_o + e\phi_{max}\left[\left(\frac{r_o}{R}\right)^p - \left(\frac{r}{R}\right)^p\right] + e\phi_{max}\left(\frac{r_o}{R}\right)^p f(2 + f) = E_o \quad ,$$

or

$$\frac{r}{R} = \frac{r_0'}{R} \sim (1 + f)^{2/p} \left(\frac{r_0}{R}\right) \quad . \tag{32}$$

If this new turning point r_0' takes the ion into a region of high magnetic field, $B = B_0(r/R_B)^m$, the perpendicular deflection of the ion by this magnetic field, $\Delta v_\perp$, changes the core radius by (see Eq. (5))

$$\frac{r_c}{R} = \frac{\Delta v_\perp}{v_\perp} \quad . \tag{33}$$

The perpendicular deflection of the ion by the magnetic field can be estimated from straightforward consideration of ion orbits in orthonormal magnetic and electric fields. Assuming that the ions were originally born at a location r_o at which magnetic deflection would give a contribution to $\Delta v_{\perp o}$ (and to r_c) of the same order of magnitude as the $\Delta v_\perp$ arising from the thermal spread $T_{\perp o}$ in the ion source, it is easy to show that the spread Δv due to a larger radial excursion to r_0', at which the magnetic field is larger ($B \sim r^m$), is

$$\frac{\Delta v(r_0')}{\Delta v(r_0)} \sim \frac{4m}{p} \frac{\delta v_r}{v_r} \approx \frac{\Delta r_c}{r_c} \quad . \tag{34}$$

Thus ion loss, in the sense that $\Delta r_c/r_c \sim 1$, requires $\delta v_r/v_r$ comparable to unity, which means a single large angle scattering. From the results in Section II, and the reactor parameters in Table I, this gives (setting $\log \Lambda = 1$)

$$\tau_{loss} - \tau_{iic} \ln\Lambda_c \frac{t_B}{t_c} - \tau_{iic} \ln\Lambda_c \left(\frac{R}{r_c}\right) - 85 \text{ ms} \quad , \tag{35}$$

which is large compared to fusion reaction times of order 10 ms, from References 1 and 2.

2. Ion Loss Due to Perpendicular Deflection in the Bulk

Perpendicular deflection due to scattering in the bulk of the device could lead to isotropization of the anisotropic ion distribution. From conservation of angular momentum (without scattering), $rv_\perp(r)$ = constant. An increase in the azimuthal ion velocity $\Delta v_\perp$ at its birth point is related to an increase in the core convergence radius, Δr_c, by

$$\frac{\Delta v_\perp(r_0)}{v_\perp(r_0)} = \frac{\Delta r_c}{r_c} \quad . \tag{36}$$

Since $v_r >> v_\perp$ through much of the ion orbit, even a small decrease in $\Delta v_r/v_r$ could produce enough $\Delta v_\perp/v_\perp \sim (\Delta v_r/v_r)(v_r/v_\perp)$ to degrade the ion focus. However, a small Δv_r would still result in a turning point at $r \simeq r_0$, where collisions would restore f_i as calculated in Section III. In order to change r_0 significantly, $\delta v_r/v_r \simeq 1$ is required, even for collisions in the bulk. The result must take into account the lower density n_B/n_c in the bulk, and the extended dwell time R/r_c in the bulk. This gives an estimate of the loss time due to collisions in the bulk of

$$\tau_{loss,B} \simeq \tau_{loss,core}(n_c/n_B)(r_c/R) \simeq \tau_{loss,core}(e\phi/E_0)^{1/2} \simeq 10 \text{ s} \quad , \tag{37}$$

for reactor parameters. This is negligible compared with τ_{fusion}.

B. Loss Due to the Edge Anisotropy

In Section III we calculated the anisotropy which develops at the edge due to small angle scattering in the rest of the device. In this section we estimate the degradation of the core which results from this anisotropy.

From Section III we showed that the increase in $T_{\perp 0}$ at $r = R$ was of order $E_0(E_0/e\phi)^{1/2}$, and that the increase in core size was $\Delta r_c/r_c \simeq \Delta v_\perp/v_\perp$. Therefore it follows that

$$\frac{\Delta r_c}{r_c} \simeq \left(\frac{E_0}{e\phi}\right)^{1/4} \leq 10^{-1} \quad , \tag{38}$$

with this small Δr_c taking place on a time scale of a few ion transits. We conclude that small angle scattering, when the Maxwellization of the plasma near the ion turning points is taken into account, is a very small contribution to core degradation, even though this is the fastest collisional effect.

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Table I. Representative SCIF parameters.

	EXP	REACTOR
$e\phi_{max}$ (depth of the potential well)	10 keV	100 keV
E_o (ion birth energy)	5 eV	5 eV
n_c (ion core density)	10^{12} cm^{-3}	10^{18} cm^{-3}
R (radius of device)	1 m	2 m
B (magnetic field at R, m = 3)	0.2 T	1 T